

MODAL ANALYSIS ON CNC MILLING CUTTING TOOL

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ABSTRACT

To date, milling is one of the most common manufacturing process for manufacturing sectors, aerospace, and tool and dies industries. In this project, the main objective is to study the dynamic properties and behaviour of the CNC milling cutting tool. The method used is using finite element analysis and the validation of results obtained using experimental modal testing. Firstly, the structural three-dimensional solid modelling of the selected CNC cutting tool draw before analysed using the linear modal analysis approach. After that, the experimental modal testing was performed using Modal Impact Hammer Testing method. The natural frequency of the mode shape is determined and comparative study was done from both method results. The results of this project shown the mode shape simulation of experimental data are totally not same but generally is in agreement with the finite element analysis. It is concluded that the experimental method uses vertical milling machines as test rig and the difference mass of cutting tool affected the result.

ABSTRAK

Sehingga kini, pengilangan adalah salah satu proses pembuatan yang paling biasa untuk sektor pembuatan, aeroangkasa, dan industri alat dan acuan. Dalam projek ini, objektif utama adalah untuk mengkaji sifat dinamik dan perilaku alat pemotong CNC. Kaedah yang digunakan adalah dengan menggunakan analisis elemen secara teori dan pengesahan keputusan yang diperolehi menggunakan ujian modal secara eksperimen. Pertama, pemodelan struktur tiga-dimensi alat pemotong CNC yang telah dipilih dilukis menggunakan perisian melukis sebelum dianalisis menggunakan pendekatan analisis linier modal. Selepas itu, ujian modal secara eksperimen telah dilakukan dengan menggunakan kaedah kesan ketukan. Frekuensi dan bentuk mod ditentukan dan kajian perbandingan telah dilakukan dari kedua-dua keputusan kaedah. Keputusan projek ini telah menunjukkan bahawa simulasi bentuk mod daripada data uji kaji secara keseluruhannya tidak sama tetapi secara umumnya adalah sama dengan simulasi daripada elemen secara teori. Ia menyimpulkan bahawa kaedah eksperimen menggunakan mesin pengilangan menegak sebagai pelantar ujian dan perbezaan jisim alat pemotong juga boleh menjejaskan hasilnya.

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LIST OF SYMBOLS

°	Degree
dB	Decibel
k	Kilo
&	And
"	inch
TiN	Titanium Nitride
TiCN	Titanium Carconitride
TiAlN	Titanium Aluminum Nitride
AlTiN	Aluminum Titanium Nitride
mm	Milimeter
N	Newton
%	Percent
volt	Voltan
Hz	Hertz
kHz	Kilohertz

LIST OF ABBREVIATIONS

CNC	Computer Numerical Control
EMA	Experimental Modal Analysis
FE	Finite Element
FFT	Fast Fourier Transform
USB	Universal Serial Bus
TCP/IP	Transmission Control Protocol/Internet Protocol
FEA	Finite Element Analysis
FEM	Finite Element Method
DOF	Degree Of Freedom
ODS	Operating Deflection Shape
IRF	Impulse Response Functions
SDOF	Single Degree Of Freedom
MDOF	Multi Degree Of Freedom
FRF	Frequency Response Function
NC	Numerical Control
HSS	High Speed Steel
DAQ	Data Acquisition
CAD	Computer Aided Diagram
2D	Two Dimensional
3D	Three Dimensional
ASCII	American Standard Code for Information Interchange
SI	International System of Units

CHAPTER 1

INTRODUCTION

1.1 GENERAL INTRODUCTION

CNC stands for Computer Numerical Control. CNC machine is a milling machine that can perform the functions of drilling and also turning. CNC milling machine has been commonly used in industrial field nowadays. CNC have various types of cutting tools and one of the common cutting tools is end mill. In CNC machines, end mills are used to cut metal and other materials. Roughing end mills used to remove large amounts of raw material to create the rough shape of the part itself. Moreover, finishing end mill used to complete and surface finished the part to size as defined in the blueprint. Vibration occurring on machine tools has been being a serious limitation for engineers since a long time ago. Undesired relative vibrations between the tool and the work-piece reduced the quality of the machine surfaces during cutting. Machine tool chatter is a self-excited vibration problem occurring in large rates of material removal, resulting from the unavoidable flexibility between the cutting tool and workpiece. In addition, chatter causes rougher surface finish and dimensional inaccuracy of the workpiece, along with unacceptably loud noise levels and accelerated tool wear. Generally, chatter is one of the most critical limiting factors, which is considered in designing a manufacturing process (Erol Turkes et al, 2010). Modal analysis is the study of the dynamic properties of structures under vibrational excitation. The dynamic behaviour of a structure in a given frequency range can be modelled as a set of individual modes of vibration. The parameters that describe each mode are natural frequency or resonance frequency (modal) damping mode shape are called the modal parameters. By using the modal parameters to model the structure, vibration problems caused by these resonances or modes can be examined and understood. To better understand any structural vibration problem, the

resonances of a structure need to be identified and quantified. A common way of doing this is to define the structure's modal parameters. Static and dynamic deformations of cutting tool play an important role in tolerance integrity and stability in a machining process affecting part quality and productivity. It is an experimental approach for solving technical problems which are a means to estimate or evaluate modal properties of a mechanical structure. Modal analysis is vital to understanding and optimizing the inherent dynamic behaviour of structures, leading to lighter, stronger, and safer structure to better performance.

In this project, we will investigate the stability and identify the vibration that occurred in the cutting tool of CNC machine. The result of vibration obtained is validate by performing dynamic analysis using ANSYS Finite Element Analysis (FEA).

1.2 OBJECTIVE OF STUDY

The purpose of this research is to study the dynamic properties and behaviour of CNC milling cutting tool by using modal analysis and comparison with the finite element analysis (FEA).

1.3 SCOPE OF PROJECT

This project focuses on the following points:

- i. Choosing and draw CNC milling cutting tool. Difficulties in modelling the cutting tool with the dimension precise with the original one.
- ii. The theoretical data for dynamic analysis using FEA ANSYS. Some problem with importing files and setting the model parameter.
- iii. Experimental analysis using modal testing on cutting tool. Difficulties in choosing the right test rig for the experiment and getting the best result.
- iv. Comparative study between numerical and experimental analysis.

1.4 PROBLEM STATEMENT

There are several factors that can potentially influence the quality of the final product of a machining. Some of these include the condition of the machine tool itself, the condition of the cutter, and the dynamics of the process. The structural dynamics of the machine tool, the dynamics of the cutting process and workpiece-tool interactions all affect the quality of the surface profile. Finite element method commonly used to analyze the instability of machining process. The tool's natural frequencies and the shape of their vibration modes were obtained from modal testing results. Thus, this project is focused on dynamic properties of a CNC cutting tool at the resonance frequencies and vibration shapes. The machine tool vibration was excited by impulse force and a response of excited vibration was recorded. The measurement points for vibration were selected at the different location of cutting tool.

The frequency of vibration of the CNC cutting tool is directly related to the stiffness and the mass of it while the mode shapes are related to the defect location. Therefore vibration testing needs to be carried out to obtain the data of those dynamic properties. The parameters that describe each mode are natural frequency or resonance frequency (modal) damping mode shape; these are called the modal parameters. By using the modal parameters to model the structure, vibration problems caused by these resonances or modes can be examined and understood. Vibration is a frequent problem that affecting the result of machining and cutting tool life. The last shape of workpiece will wavy surface during machining operation. This is showing the vibration problem occurs and affects the surface finish. Before going further on optimization of cutting tool or active suppression on chatter, initial study on the structural analysis need to be conducted.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter is reviewed about the preliminary of vibration in the CNC machine cutting tool, the dynamic properties and structural behaviour with the cutting tool using different type of cutting tool. Modal analysis or vibration analysis was done using two methods that is by experimental of modal analysis and simulation using ANSYS finite element analysis.

2.2 MODAL ANALYSIS OF MILLING MACHINE

This chapter aims to investigate the vibration phenomena occurring occasionally at the different components of milling machine. Moreover, it will involve the previous analytical and experimental modal analyses performed. The study focused on extracting the mode shape of the dominating cutting tools of the milling machine in order to ensure resonance phenomena as a cause of chatter. In a first step the significant eigen-frequencies with corresponding mode shapes were obtained by means of an experimental modal analysis (EMA). Subsequently, the dynamic behaviour of the machine components was simulated using an ABAQUS FE model by Anayet U. Patrawi et al., 2009. However, ANSYS FE model is used in this project. The comparison of the eigenfrequencies based on FE calculations with their experimental counterparts proved in general quite a satisfactory correlation. (Anayet U. Patrawi et al., 2009)

2.2.1 Experimental Modal Analysis

Understanding of experimental modal analysis and knows how this method works on this project can give explanation to investigate the way to get dynamic properties that is modal parameter. Figure 2.1 below shown how is the experimental modal analysis test setup.

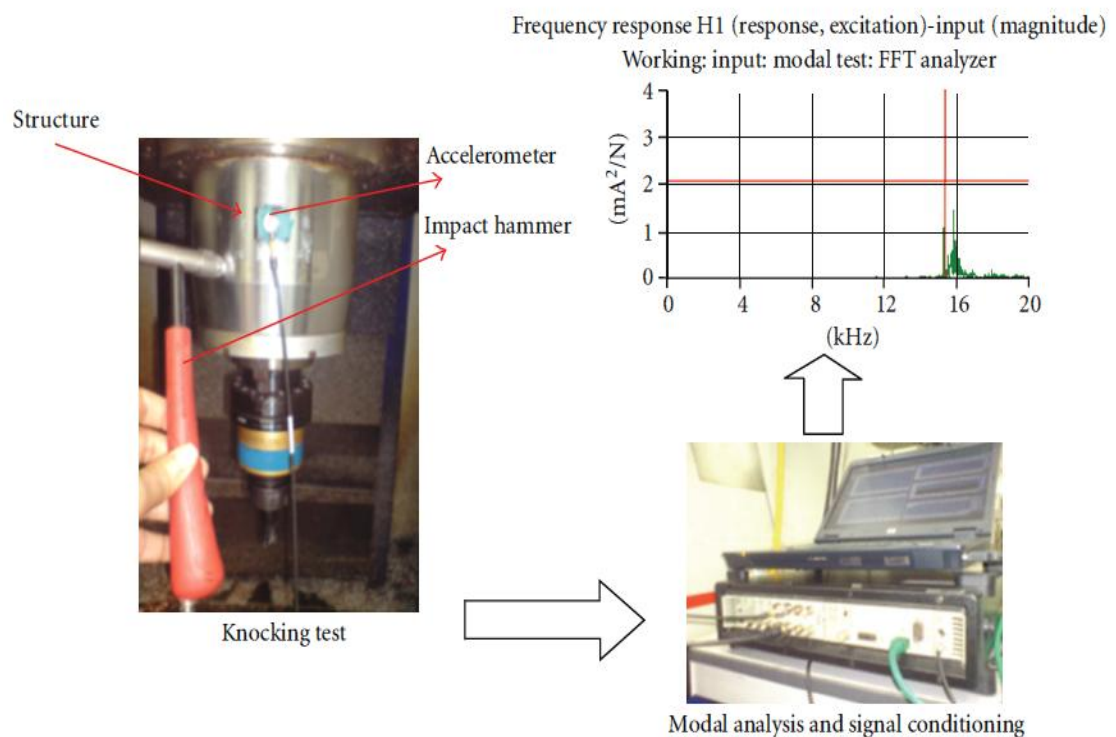


Figure 2.1: Test Set Up

Source: Anayet U. Patrawi et al., 2009

(a) Measurement Hardware.

A vibration measurement generally requires several hardware components. The basic hardware elements required consist of a source of excitation, called an exciter (Impulse hammer), for providing a known or controlled input force to the structure, a transducer to convert the mechanical motion of the structure into an electrical signal, a signal conditioning amplifier, and an analysis system in which

modal analysis program resides. The schematic diagram of hardware used to perform in a vibration test is shown in Figure 2.1. The different equipments that have been used are listed as follows: Pulse Front-end (Data Acquisition), Impact Hammer, USB Dongle, Accelerometers, Impact Hammer cable, Accelerometer cables, Pulse Front-End Power Supply, TCP/IP Cross Cable, and Bee's wax.(Anayet U. Patrawi et al., 2009)

(b) Test Procedures.

The different milling machine components were identified which play a dominating role in the chatter generation. The natural frequency of the different components was measured using modal analysis and consequently the different mode shapes were identified. Initially excited frequencies were monitored during the operational mode. It is easy to record a response in vibration during machining but almost impossible to measure the mentioned dynamic force. Therefore, the force measurement was replaced by measurement of the impulse response to the impact force excited by a hammer, whose tip was fitted with a force sensor. As the goal of these measurements was to evaluate frequency transfer function, the responses at various machine points with respect to a reference point were recorded and analysed. The reference point was selected at the different location shown in Figure 2.2. (Anayet U. Patrawi et al., 2009)

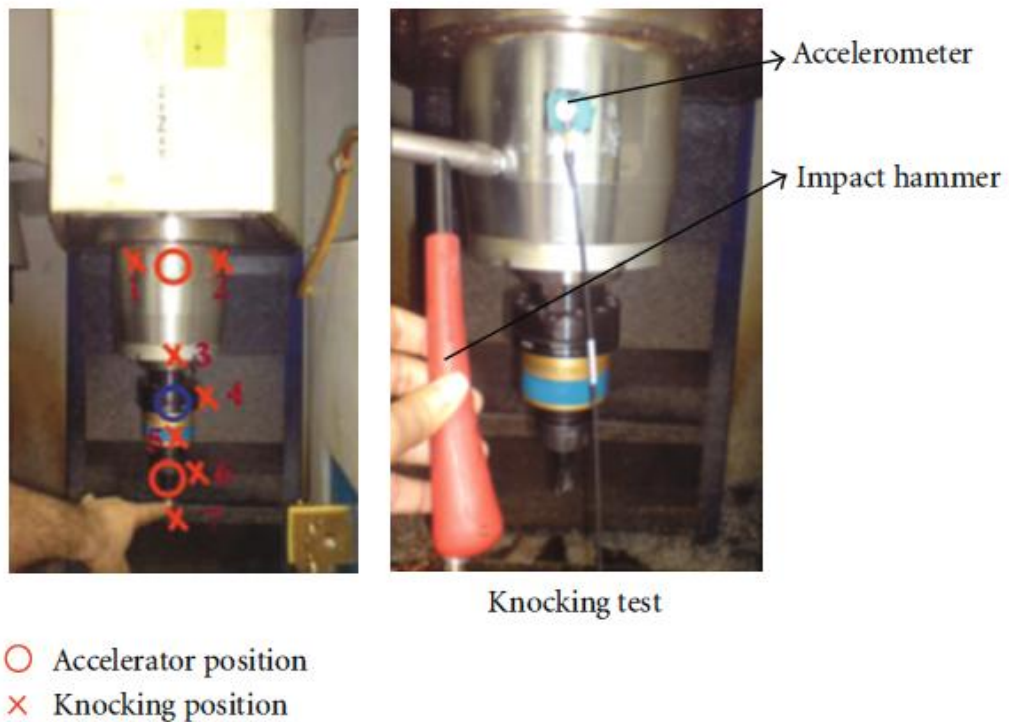


Figure 2.2: Accelerometer and knocking point position

Source: Anayet U. Patrawi et al., 2009

(i) Knocking test.

The natural frequencies of the different components were extracted from the recorded FFT diagram. One accelerometer was connected to the component; the natural frequency data from the FFT graph was recorded by knocking the different components using the impact hammer. (Anayet U. Patrawi et al., 2009)

Experimental Modal Analysis is based on determining the modal parameters by testing, unlike Analytical Modal Analysis, where the modal parameters are derived from Finite Element Models (FEMs). There are two ways of doing Experimental Modal Analysis: Classical Modal Analysis and Advances in Acoustics and Vibration. (Anayet U. Patrawi et al., 2009)

2.3 MACHINING PROCESS DYNAMICS

The machining process is considered from the perspective of complex interrelated dynamics of the machine tool mechanical structure and workpiece-cutting interactions. This approach was initially proposed by Merritt HE, 1965; Kegg RL, 1965; Minis et al., 1990; and Tlustý and Moriwaki T, 1976 and was extended by Bordatchev EV and Orban PE, 1999 to deal with the dynamics of the machine tool spindle only. The concepts expand upon in this paper to include the dynamics of the cutting tool and thus create a more complete dynamic model of this important machine tool subsystem. (Adam G. Rehorn et al., 2004)

2.3.1 Variables Influencing The Machining Process

A simple way of envisioning and discussing machining is to consider the process as a black box (Rehorn AG, 2001; Bordatchev and Orban, 1999). There are three main parts associated with this model: the input variables, the output variables and the process itself. The output of the machining operation is a workpiece with a given surface profile and geometry. This surface profile can be quantified by a product quality vector (Bordatchev and Orban, 1999). The quality vector includes measurements of ovality, cylindricity and dimensional accuracy as well as some quantification of the roughness of the surface finish. The main input variables are of two types: controlled variables and characteristic variables. The controlled variables represent factors that can be independently controlled and altered by the operator of the machine tool, either before or at any time during machining. Controlled variables include spindle speed and feed rate, the cutting and feed motions, depth of cut, cut geometry, immersion and direction of rotation of the tool (Altintas Y, 2000). Characteristic variables are those over which there is little or no control once selected and which are primarily process specific. Some typical examples include the cutter type and geometry, the physical and mechanical properties of the workpiece and the cutting tool and the chemical affinities between these two materials. Even though very little control can be exercised over the characteristic variables, they can have a dramatic impact on the quality of the product. (Adam G. Rehorn et al., 2004)

The machining process itself can be considered as a series of interactions among several dynamic processes. These processes represent different sets of dynamics that comprise the constituent parts of the machining process. Some of these dynamics, namely the machine tool's natural structural dynamics and the dynamics of the cutting process, result in workpiece-tool relative displacements. These displacements then affect the dynamics of the surface profile formation. Thus, the dynamics can be seen to directly influence the quality of the final product. .(Adam G. Rehorn et al., 2004)

2.3.2 Interrelation Among Different Variables In Machining Processes

The machining process and the interrelations among the dynamic processes mentioned in the previous section can be considered in terms of a dynamic feedback system, which is a further development of previously presented models (Bordatchev,1996; Zakovorotny et al.,1995). Each block represents a key dynamic process in the overall machining operation. The dynamic processes are represented as functions of the differential operator.(Adam G. Rehorn et al., 2004)

2.4 FUNDAMENTAL OF VIBRATION

2.4.1 Natural Frequencies

Bridges, aircraft wings, machine tools, and all other physical structures have natural frequencies. A natural frequency is the frequency at which the structure would oscillate if it were disturbed from its rest position and then allowed to vibrate freely (Tom Irvine,2000). All structures have at least one natural frequency. Nearly every structure has multiple natural frequencies. Resonance occurs when the applied force or base excitation frequency coincides with structural natural frequency. During resonant vibration, the response displacement may increase until the structure experiences buckling, yielding, fatigue, or some other failure mechanism.(Tom Irvine,2000)

(a) Dynamic Analysis

Engineers performing dynamic analysis must:

1. Determine the natural frequencies of the structure.
2. Characterize potential excitation functions.
3. Calculate the response of the structure to the maximum expected excitation.
4. Determine whether the expected response violates any failure criteria.

Based on the steps above, this report related to the first step. The natural frequencies can be calculated via analytical methods during the design stage. The frequencies may also be measured after the structure, or a prototype, is built. Each natural frequency has a corresponding damping ratio. Damping values are empirical values that must be obtained by measurement. (Tom Irvine,2000)

(b) Frequency Response Function Overview

There are many tools available for performing vibration analysis and testing. The frequency response function is a particular tool. A frequency response function (FRF) is a transfer function, expressed in the frequency domain. Frequency response functions are complex functions, with real and imaginary components. They may also be represented in terms of magnitude and phase. A frequency response function can be formed from either measured data or analytical functions. A frequency response function expresses the structural response to an applied force as a function of frequency. The response may be given in terms of displacement, velocity, or acceleration. Furthermore, the response parameter may appear in the numerator or denominator of the transfer function. (Tom Irvine,2000)

(c) FRF Model

Consider a linear system as represented by the diagram in Figure 2.3:

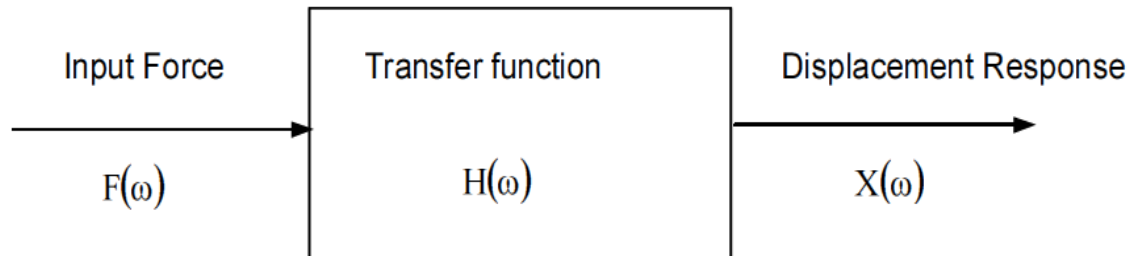


Figure 2.3: linear system of FRF

Source: Tom Irvine, 2000

The function above is a complex function and also can be represented in terms of phase and magnitude. $F(\omega)$ is the input force in function of angular frequency ω , $H(\omega)$ is the transfer function and $X(\omega)$ is the displacement response function.

(d) FRF Measurements

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of a mechanical structure. Experimental modal parameters (frequency, damping, and mode shape) are also obtained from a set of FRF measurements. The FRF describes the input-output relationship between two points on a structure as a function of frequency, as shown in Figure 2.3. Since both force and motion are vector quantities, they have directions associated with them. Therefore, an FRF is actually defined between a single input DOF (point & direction), and a single output DOF. An FRF is a measure of how much displacement, velocity, or acceleration response a structure has at an output DOF, per unit of excitation force at an input DOF. (Brian J. Schwarz et al, 1999)

Figure 2.4 shows that an FRF is defined as the ratio of the Fourier transform of an output response $X(\omega)$ divided by the Fourier transform of the input force $F(\omega)$ that caused the output.

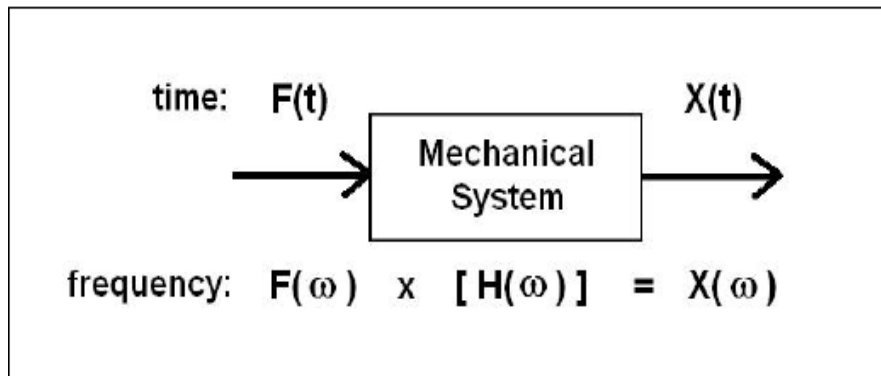


Figure 2.4: Block Diagram of FRF

Source: Brian J. Schwarz et al,1999

Response motion can be measured as displacement, velocity, or acceleration, and the FRF and its inverse can have a variety of names such as,

- i. Compliance (displacement / force)
- ii. Mobility (velocity / force)
- iii. Inertance or Receptance (acceleration / force)
- iv. Dynamic Stiffness (1 / Compliance)
- v. Impedance (1 / Mobility)
- vi. Dynamic Mass (1 / Inertance)

2.4.2 Modes

Modes (or resonances) are inherent properties of a structure. Resonances are determined by the material properties (mass, stiffness, and damping properties), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape. If either the material properties or the boundary conditions of a structure change, its modes will change.